Problem 48

A particle undergoes three consecutive displacements given by vectors $\vec{\mathbf{D}}_1 = (3.0\hat{\mathbf{i}} - 4.0\hat{\mathbf{j}} - 2.0\hat{\mathbf{k}})$mm, $\vec{\mathbf{D}}_2 = (1.0\hat{\mathbf{i}} - 7.0\hat{\mathbf{j}} + 4.0\hat{\mathbf{k}})$mm, and $\vec{\mathbf{D}}_3 = (-7.0\hat{\mathbf{i}} + 4.0\hat{\mathbf{j}} + 1.0\hat{\mathbf{k}})$mm. (a) Find the resultant displacement vector of the particle. (b) What is the magnitude of the resultant displacement? (c) If all displacements were along one line, how far would the particle travel?

Solution

Part (a)

The resultant displacement vector of the particle is the sum of the three given displacement vectors.

$$\begin{aligned}\vec{\mathbf{D}}_{\text{resultant}} &= \vec{\mathbf{D}}_1 + \vec{\mathbf{D}}_2 + \vec{\mathbf{D}}_3 \\ &= (3.0\hat{\mathbf{i}} - 4.0\hat{\mathbf{j}} - 2.0\hat{\mathbf{k}})\text{mm} + (1.0\hat{\mathbf{i}} - 7.0\hat{\mathbf{j}} + 4.0\hat{\mathbf{k}})\text{mm} + (-7.0\hat{\mathbf{i}} + 4.0\hat{\mathbf{j}} + 1.0\hat{\mathbf{k}})\text{mm} \\ &= (3.0 + 1.0 - 7.0)\hat{\mathbf{i}}\ \text{mm} + (-4.0 - 7.0 + 4.0)\hat{\mathbf{j}}\ \text{mm} + (-2.0 + 4.0 + 1.0)\hat{\mathbf{k}}\ \text{mm} \\ &= (-3.0\hat{\mathbf{i}} - 7.0\hat{\mathbf{j}} + 3.0\hat{\mathbf{k}})\text{mm}\end{aligned}$$

Part (b)

Calculate the magnitude.

$$\begin{aligned}\left|\vec{\mathbf{D}}_{\text{resultant}}\right| &= \sqrt{(-3.0\ \text{mm})^2 + (-7.0\ \text{mm})^2 + (3.0\ \text{mm})^2}\ \text{mm} \\ &= \sqrt{67\ \text{mm}^2} \\ &\approx 8.19\ \text{mm}\end{aligned}$$

Part (c)

Suppose all displacements are along one line. The resultant displacement in this case is

$$\begin{aligned}\vec{\mathbf{D}}_{\text{resultant}} &= (3.0\hat{\mathbf{i}} - 4.0\hat{\mathbf{i}} - 2.0\hat{\mathbf{i}})\text{mm} + (1.0\hat{\mathbf{i}} - 7.0\hat{\mathbf{i}} + 4.0\hat{\mathbf{i}})\text{mm} + (-7.0\hat{\mathbf{i}} + 4.0\hat{\mathbf{i}} + 1.0\hat{\mathbf{i}})\text{mm} \\ &= (3.0 - 4.0 - 2.0 + 1.0 - 7.0 + 4.0 - 7.0 + 4.0 + 1.0)\hat{\mathbf{i}}\ \text{mm} \\ &= (-7\ \text{mm})\hat{\mathbf{i}},\end{aligned}$$

which means the particle would be 7 millimeters from its starting location.